

Dynamical Systems and Accelerator Theory Group

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<http://www.physics.umd.edu/dsat/>

Maryland People

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General Description of Work

The motion of a charged particle in an electromagnetic field is governed by a Hamiltonian. Hamiltonians generate symplectic maps. Consequently, the time evolution of particles in an accelerator is described by a symplectic map. Symplectic maps in turn form an infinite-dimensional Lie group. An extensive calculus, involving Lie-algebraic tools, has been developed for the computation, manipulation, and analysis of symplectic maps including high-order nonlinear effects. These methods are now coming into routine use in the design and operation of accelerators including the Stanford B factory, the Large Hadron Collider under construction at CERN, the proposed Next Linear Collider, as well as various Synchrotron Light Sources.

Current Efforts

1. Documentation

- a. * MaryLie 3.0 Manual (some 900 pages including an index).
- b. * Lie Algebraic Methods for Nonlinear Dynamics (900 pages and growing).

2. Algorithm Development

- a. * Higher-order map generation and concatenation formulas. (Dragt and Cooper)
- b. * Canonical treatment of translations: Embedding non-origin preserving maps in 6-dimensional phase space in the set of origin preserving maps in 8-dimensional phase space. (Dragt)
- c. * Computing Lie maps for real beamline-elements using numerically produced surface field data. [Dragt, Mitchell, and Walstrom (LANL)]
- d. Computing Taylor maps for non-Hamiltonian systems. (Dragt)

3. Algorithm Implementation

- a. * MaryLie 7.1. [Dragt, Cornick, Cooper, and Mottershead (LANL retired)]
- b. * Item 2c above. (Dragt and Mitchell)
- c. Taylor map for driven Duffing equation; comparison of Feigenbaum diagrams for stroboscopic map computed numerically and from Taylor map. (Dragt and Papadopoulos)

4. Theory Development

- a. Time Reversal Properties of Hamiltonians and their Transfer Maps. (Dragt)
- b. Normal Form and Transfer Maps for Morse Oscillator. (Dragt and Strauch)

5. Fundamental Studies

- a. * Numerical Propagation of Charged-Particle Wave Packets in Electromagnetic Fields with the Goal of Deducing from First principles [Quantum Electrodynamics, (QED)] Quantum and Synchrotron Radiation Effects. (Dragt and Johnson)
- b. Quantum Computing and Split-Operator Methods. (Dragt, Johnson, and Strauch)

Further Discussion of * Items

A. Documentation

B. Higher-order map generation and concatenation formulas

C. Canonical treatment of translations: Embedding non-origin preserving maps in 6-dimensional phase space in the set of origin preserving maps in 8-dimensional phase space

Note: Implementation of items B and C above is one of the major tasks for constructing MaryLie 7.1.

Map Generation Formulas (Dimension Independent)

$$\mathcal{M} = \exp(:f_2^c:) \exp(:f_2^a:) \exp(:f_3:) \exp(:f_4:) \dots$$

$$\mathcal{M}_2 = \exp(:f_2^c:) \exp(:f_2^a:)$$

$$\dot{R}=JSR.$$

$$H_m^{\text{int}}(\zeta^i,t) = H_m(\mathcal{M}_2\zeta^i,t).$$

$$\dot{f}_3=-H_3^{\text{int}},$$

$$\dot{f}_4=-H_4^{\text{int}}+(:f_3:/2)(-H_3^{\text{int}}),$$

$$\dot{f}_5=-H_5^{\text{int}}+ :f_3:(-H_4^{\text{int}})+(1/3):f_3:^2(-H_3^{\text{int}}),$$

$$\begin{aligned}\dot{f}_6= & -H_6^{\text{int}}+ :f_3:(-H_5^{\text{int}})+(1/2):f_4:(-H_4^{\text{int}}) \\ & +(1/4):f_4::f_3:(-H_3^{\text{int}})+(1/2):f_3:^2(-H_4^{\text{int}}) \\ & +(1/8):f_3:^3(-H_3^{\text{int}}),\end{aligned}$$

$$\begin{aligned}\dot{f}_7 = & - H_7^{\text{int}} + : f_3 : (-H_6^{\text{int}}) + : f_4 : (-H_5^{\text{int}}) + : f_4 :: f_3 : (-H_4^{\text{int}}) \\ & + (1/3) : f_4 :: f_3 :^2 (-H_3^{\text{int}}) + (1/2) : f_3 :^2 (-H_5^{\text{int}}) \\ & + (1/6) : f_3 :^3 (-H_4^{\text{int}}) + (1/30) : f_3 :^4 (-H_3^{\text{int}}),\end{aligned}$$

$$\begin{aligned}\dot{f}_8 = & - H_8^{\text{int}} + : f_3 : (-H_7^{\text{int}}) + : f_4 : (-H_6^{\text{int}}) + : f_4 :: f_3 : (-H_5^{\text{int}}) \\ & + (1/2) : f_4 :: f_3 :^2 (-H_4^{\text{int}}) + (1/8) : f_4 :: f_3 :^3 (-H_3^{\text{int}}) \\ & + (1/2) : f_5 : (-H_5^{\text{int}}) + (1/2) : f_5 :: f_3 : (-H_4^{\text{int}}) \\ & + (1/6) : f_5 :: f_3 :^2 (-H_3^{\text{int}}) + (1/2) : f_3 :^2 (-H_6^{\text{int}}) \\ & + (1/3) : f_4 :^2 (-H_4^{\text{int}}) + (1/6) : f_4 :^2 : f_3 : (-H_3^{\text{int}}) \\ & + (1/6) : f_3 :^3 (-H_5^{\text{int}}) + (1/24) : f_3 :^4 (-H_4^{\text{int}}) \\ & + (1/144) : f_3 :^5 (-H_3^{\text{int}}),\end{aligned}$$

$$\text{Concatenation Formulas (Dimension Independent)}$$

$$\mathcal{M}_f = \exp(:f^c_2:) \exp(:f^a_2:) \exp(:f_3:) \exp(:f_4:) \cdots$$

$$\mathcal{M}_g = \exp(:g^c_2:) \exp(:g^a_2:) \exp(:g_3:) \exp(:g_4:) \cdots$$

$$\mathcal{M}_h=\mathcal{M}_f\mathcal{M}_g$$

$$\mathcal{M}_h = \exp(:h^c_2:) \exp(:h^a_2:) \exp(:h_3:) \exp(:h_4:) \cdots$$

$$R^h=R^g R^f$$

$$f_m^{tr}(z)=f_m[(R^g)^{-1}z],$$

$$\begin{aligned} h_3 &= f_3^{tr} + g_3, \\ h_4 &= f_4^{tr} + g_4 + [f_3^{tr}, g_3]/2 \end{aligned}$$

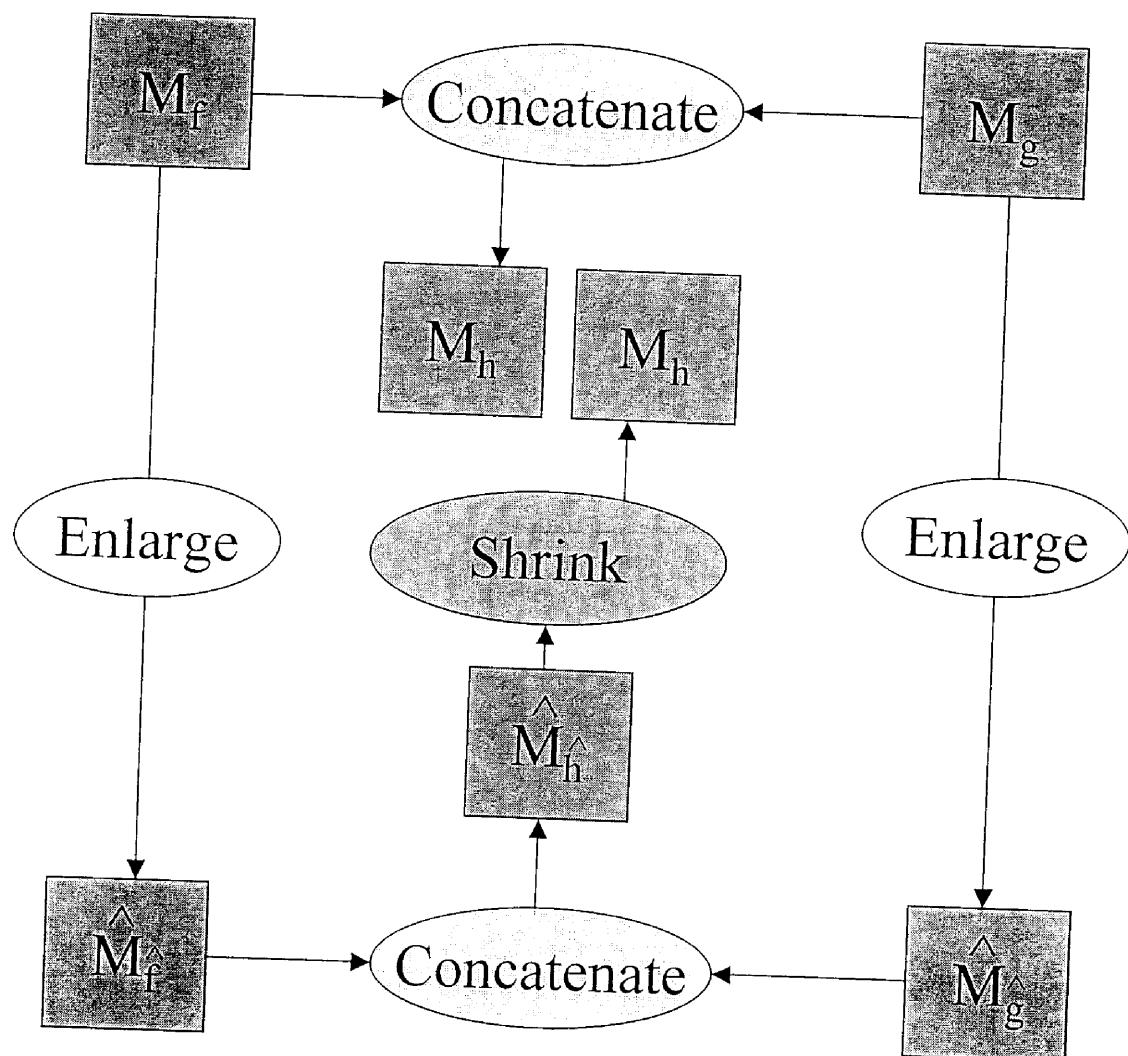
$$h_5 = f_5^{tr} + g_5 - [g_3, f_4^{tr}] + \frac{1}{3} : g_3 :^2 f_3^{tr} - \frac{1}{6} : f_3^{tr} :^2 g_3,$$

$$\begin{aligned} h_6 &= f_6^{tr} + g_6 - [g_3, f_5^{tr}] + \frac{1}{2} : g_3 :^2 f_4^{tr} + \frac{1}{2} [f_4^{tr}, g_4] - \frac{1}{4} [f_4^{tr}, [f_3^{tr}, g_3]] \\ &- \frac{1}{4} [g_4, [f_3^{tr}, g_3]] + \frac{1}{24} : f_3^{tr} :^3 g_3 - \frac{1}{8} : g_3 :^3 f_3^{tr} \\ &+ \frac{1}{8} [f_3^{tr}, [g_3, [f_3^{tr}, g_3]]], \end{aligned}$$

$$\begin{aligned} h_7 &= f_7^{tr} + g_7 - : g_3 : f_6^{tr} - : g_4 : f_5^{tr} + \frac{1}{2} : g_3 :^2 f_5^{tr} \\ &+ \frac{1}{2} : f_4^{tr} :: g_3 : f_4^{tr} + : g_4 :: g_3 : f_4^{tr} + \frac{1}{3} : g_3 :: f_4^{tr} :: f_3^{tr} : g_3 \\ &- \frac{1}{6} : g_3 :^3 f_4^{tr} - \frac{1}{6} : g_4 :: f_3^{tr} :: g_3 : f_3^{tr} - \frac{1}{6} : f_4^{tr} :: f_3^{tr} :: g_3 : f_3^{tr} \\ &- \frac{1}{3} : g_4 :: g_3 :^2 f_3^{tr} - \frac{1}{3} [: g_3 : f_3^{tr}, : g_3 : f_4^{tr}] \\ &- \frac{1}{120} : f_3^{tr} :^4 g_3 - \frac{1}{30} : g_3 :: f_3^{tr} :^3 g_3 - \frac{1}{20} : g_3 :^2 f_3^{tr} :^2 g_3 \\ &+ \frac{1}{30} : g_3 :^4 f_3^{tr} + \frac{1}{30} [: f_3^{tr} : g_3, : f_3^{tr} :^2 g_3] + \frac{1}{15} [: g_3 : f_3^{tr}, : g_3 :^2 f_3^{tr}], \end{aligned}$$

$$\begin{aligned}
h_8 = & f_8^{tr} + g_8 - :g_3:f_7^{tr} - :g_4:f_6^{tr} - \frac{1}{2} :g_5:f_5^{tr} \\
& + \frac{1}{2} :g_3:^2 f_6^{tr} + \frac{1}{2} :f_5^{tr} :: g_3:f_4^{tr} + :g_4 :: g_3:f_5^{tr} + \frac{1}{2} :g_5 :: g_3:f_4^{tr} + \frac{1}{3} :g_4:^2 f_4^{tr} \\
& + \frac{1}{6} :f_4^{tr} :: g_4:f_4^{tr} - \frac{1}{6} :g_3:^3 f_5^{tr} - \frac{1}{12} :f_5^{tr} :: f_3^{tr} :: g_3:f_3^{tr} - \frac{1}{6} :g_3 :: f_5^{tr} :: g_3:f_3^{tr} \\
& - \frac{1}{12} :g_5 :: f_3^{tr} :: g_3:f_3^{tr} - \frac{1}{6} :g_5 :: g_3:^2 f_3^{tr} - \frac{1}{6} [:g_3:f_3^{tr},:g_3:f_5^{tr}] \\
& - \frac{1}{6} :f_4^{tr}:^2 g_3:f_3^{tr} - \frac{1}{4} :g_3 :: f_4^{tr} :: g_3:f_4^{tr} - \frac{1}{3} :g_4 :: f_4^{tr} :: g_3:f_3^{tr} \\
& - \frac{1}{2} :g_4 :: g_3:^2 f_4^{tr} - \frac{1}{6} [:g_3:f_3^{tr},:g_4:f_4^{tr}] - \frac{1}{6} :g_4:^2 g_3:f_3^{tr} \\
& + \frac{1}{24} :f_4^{tr} :: f_3^{tr}:^2 g_3:f_3^{tr} + \frac{1}{8} :g_3 :: f_4^{tr} :: f_3^{tr} :: g_3:f_3^{tr} + \frac{1}{8} :g_3:^2 f_4^{tr} :: g_3:f_3^{tr} \\
& + \frac{1}{24} :g_3:^4 f_4^{tr} - \frac{1}{24} [:g_3:f_4^{tr},:f_3^{tr} :: g_3:f_3^{tr}] - \frac{1}{12} [:g_3:f_4^{tr},:g_3:^2 f_3^{tr}] \\
& + \frac{1}{24} [:g_3:f_3^{tr},:f_4^{tr} :: g_3:f_3^{tr}] + \frac{1}{8} [:g_3:f_3^{tr},:g_3:^2 f_4^{tr}] + \frac{1}{24} :g_4 :: f_3^{tr}:^2 g_3:f_3^{tr} \\
& + \frac{1}{8} :g_4 :: g_3 :: f_3^{tr} :: g_3:f_3^{tr} + \frac{1}{8} :g_4 :: g_3:^3 f_3^{tr} + \frac{1}{24} [:g_3:f_3^{tr},:g_4 :: g_3:f_3^{tr}] \\
& - \frac{1}{720} :f_3^{tr}:^4 g_3:f_3^{tr} - \frac{1}{144} :g_3 :: f_3^{tr}:^3 g_3:f_3^{tr} - \frac{1}{144} [:g_3:f_3^{tr},:f_3^{tr}:^2 g_3:f_3^{tr}] \\
& - \frac{1}{72} :g_3:^2 f_3^{tr}:^2 g_3:f_3^{tr} - \frac{1}{48} [:g_3:f_3^{tr},:g_3 :: f_3^{tr} :: g_3:f_3^{tr}] - \frac{1}{72} :g_3:^3 f_3^{tr} :: g_3:f_3^{tr} \\
& - \frac{1}{48} [:g_3:f_3^{tr},:g_3:^3 f_3^{tr}] - \frac{1}{144} :g_3:^5 f_3^{tr}.
\end{aligned}$$

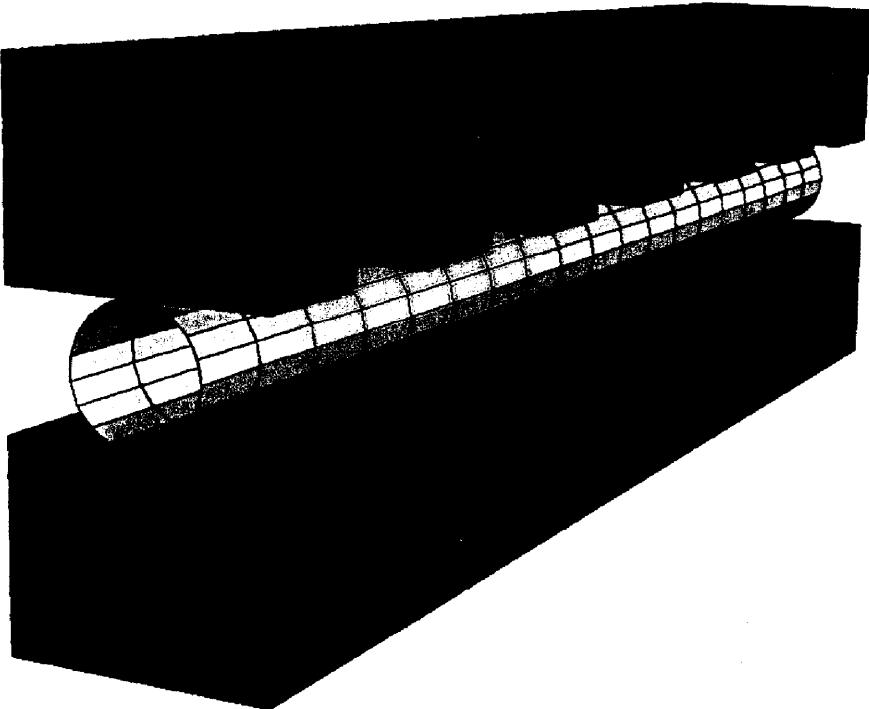
Canonical treatment of translations: Embedding non-origin preserving maps in 6-dimensional phase space in the set of origin preserving maps in 8-dimensional phase space.
 Shrinker can be constructed using concatenator.



D. Computing Lie maps for real beamline-elements using numerically produced surface field data. Example of Cornell wiggler.

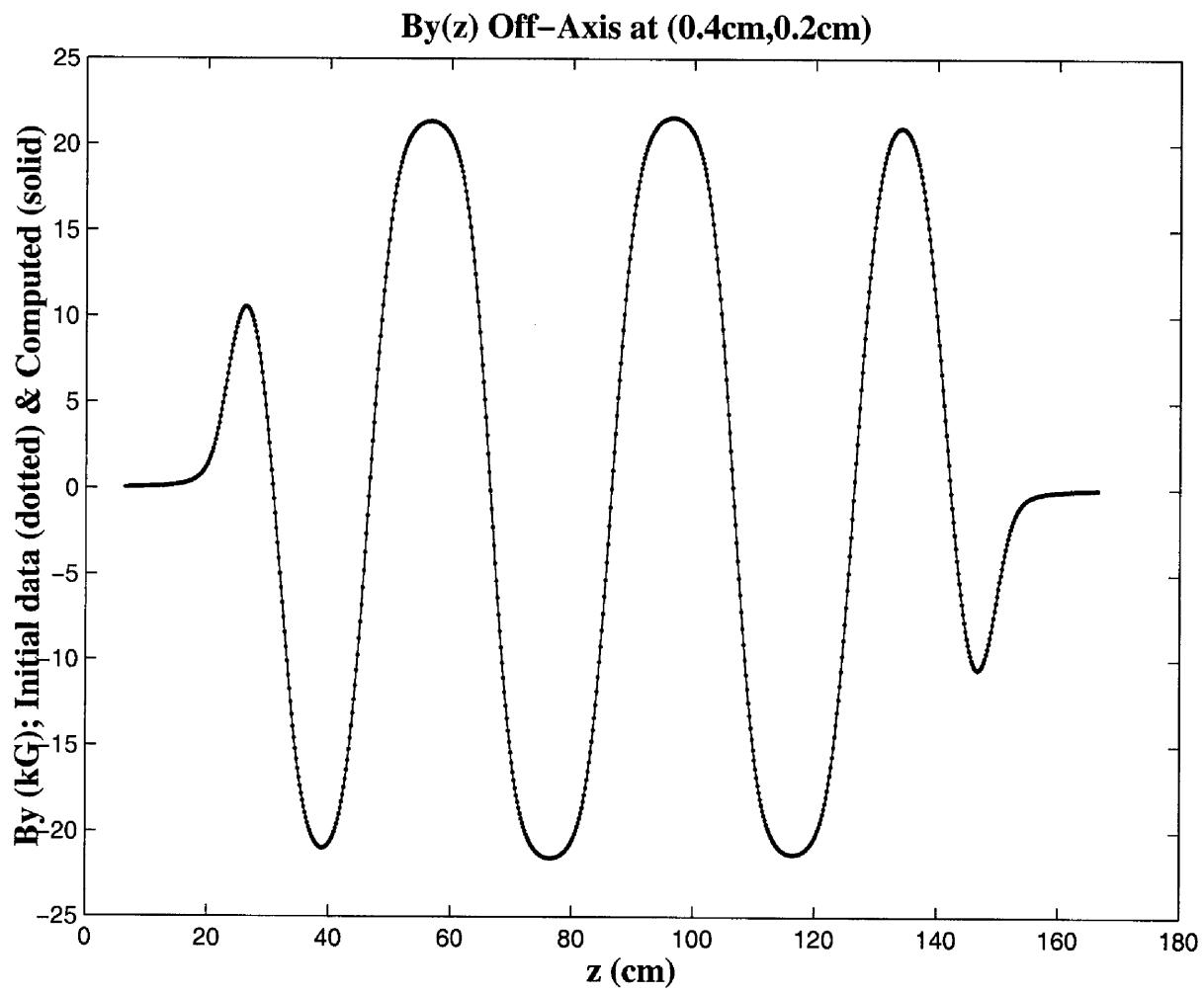
|| Fit the field away from the axis

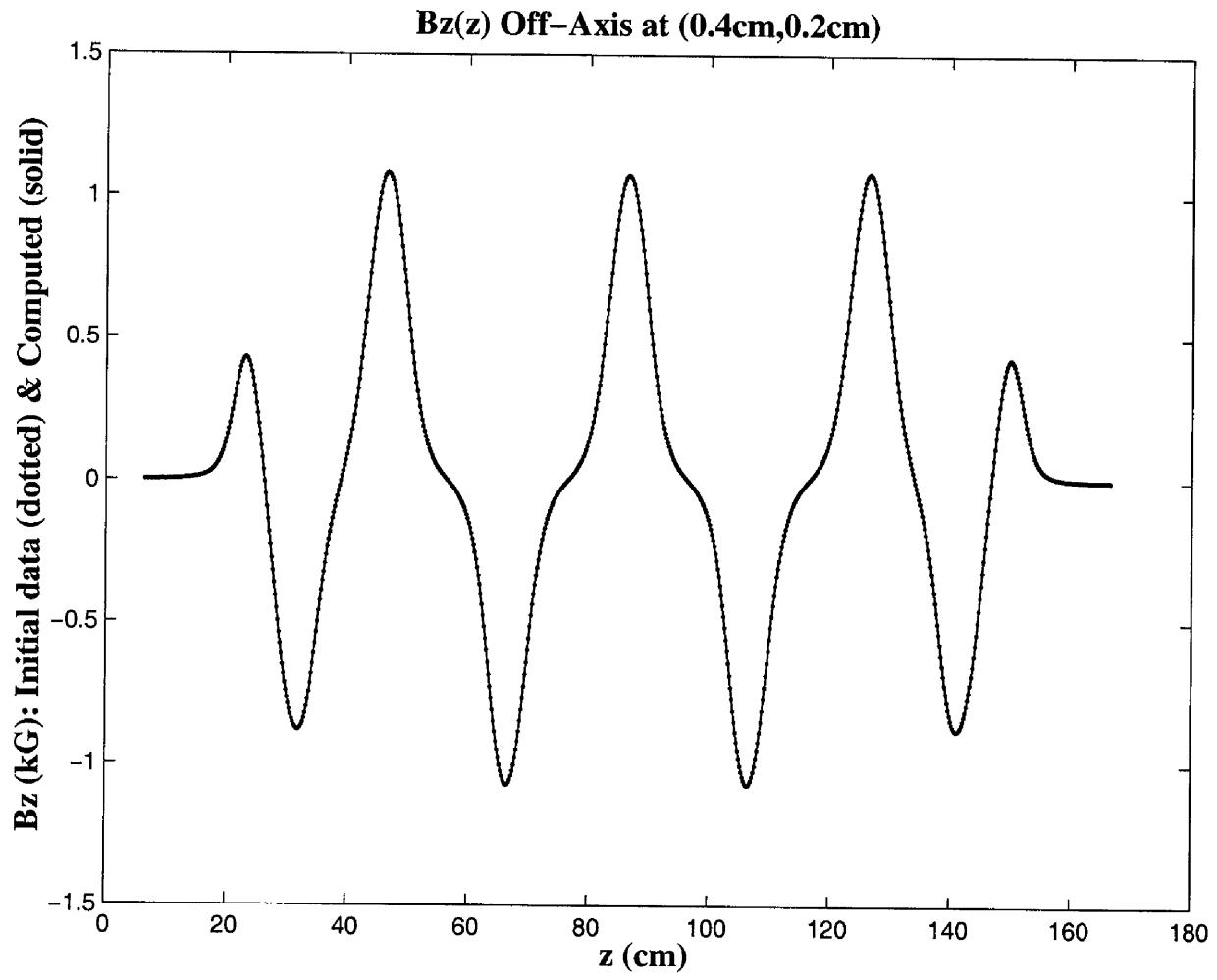
to improve accuracy

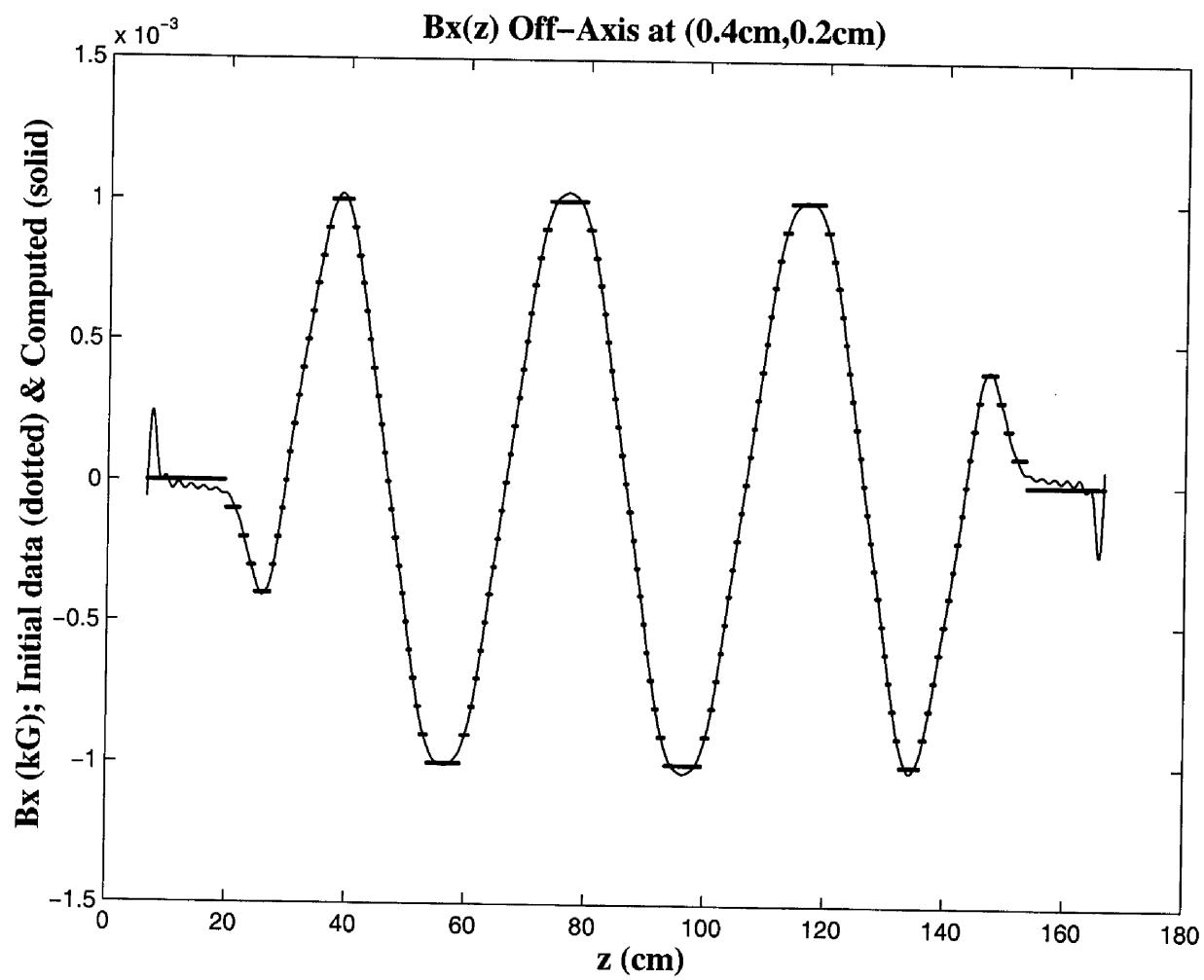


Circular Cylinder --> Bessel Functions

Elliptical Cylinder --> Mathieu Functions







E. Numerical Propagation of Charged-Particle Wave Packets in Electromagnetic Fields and Synchrotron Radiation Effects